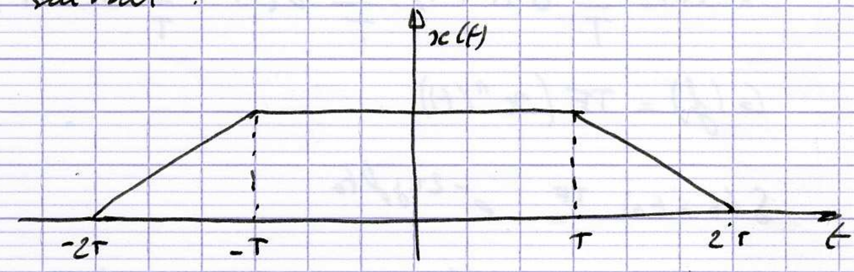
Cas suivant.



étapes :

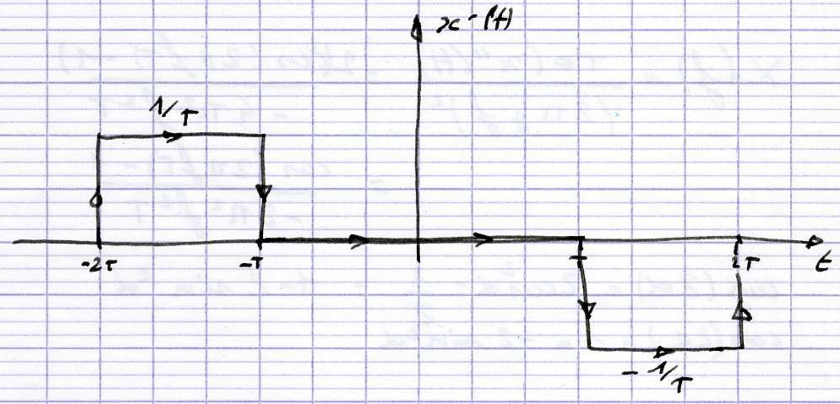
① Calculer $x'(t)$ graphiquement

② — — $x''(t)$ — — — —

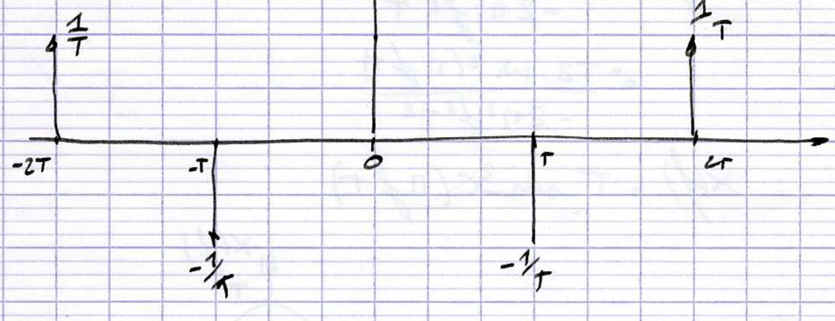
③ en déduire $G(f) = TF(x''(t))$

④ Retrouver $X(f)$

①



②



$$x''(t) = \frac{1}{T}\,\delta(t+2T) - \frac{1}{T}\,\delta(t+T) - \frac{1}{T}\,\delta(t-T) + \frac{1}{T}\,\delta(t-2T)$$

$G(f) = TF(x''(t))$

$\delta(t - t_0) \xrightarrow{TF} e^{-2\pi j f t_0}$

$$G(f) = \frac{1}{T} e^{2\pi j f(2T)} - \frac{1}{T} e^{2\pi j f(T)} - \frac{1}{T} e^{2\pi j f(-T)} + \frac{1}{T} e^{2\pi j f(-2T)}$$

$$= \frac{1}{T}\left(e^{2\pi j f(2T)} + e^{2\pi j f(-2T)}\right) - \frac{1}{T}\left(e^{2\pi j f(T)} + e^{2\pi j f(T)}\right)$$

$$= \frac{1}{T}\left(e^{2\pi j f(2T)} + e^{-2\pi j f(2T)}\right) - \frac{1}{T}\left(e^{2\pi j f(T)} + e^{-2\pi j f(T)}\right)$$

$$= \frac{1}{T}\left(2\cos(2\pi f(2T))\right) - \frac{1}{T}\left(2\cos(2\pi f(T))\right)$$

$$= \frac{1}{T}\left(2\cos(4\pi f T) - 2\cos(2\pi f T)\right)$$

$\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right) \cdot \sin\left(\frac{a-b}{2}\right)$

$$= \frac{-4}{T}\left[\sin\left(\frac{6\pi f T}{2}\right) \times \sin\left(\frac{2\pi f T}{2}\right)\right]$$

$$= \frac{-4}{T}\left[\sin(3\pi f T) \times \sin(\pi f T)\right]$$

$$= \frac{-4}{T}\left[\sin(\pi f T) \times \sin(\pi f T)\right] = \frac{-4}{T}\sin^2(\pi f T)$$

$$X(f) = \frac{TF(x''(t))}{(2\pi j f)^2}$$

$$= \frac{-4}{T}\left(\frac{\sin^2(\pi f T)}{-4\pi^2 f^2}\right) = \frac{-1}{T}\,\frac{\sin^2(\pi f T)}{-8\pi^2 f^2}$$

$$= \frac{-T \sin^2(\pi f T)}{-8\pi^2 f^2 T^2} = \frac{T}{8} \times \frac{\sin^2(\pi f T)}{(\pi f T)^2}$$

$$= \frac{T}{8}\,\mathrm{sinc}^2(\pi f T)$$